15. e^x , $\sqrt{1+x^4}$ – same as x^2

$$\cancel{\lim_{x\to\infty} \frac{\sqrt{1+x^4}}{e^x} = \lim_{x\to\infty} \frac{4x^3 \cdot \frac{1}{2}(1+x^4)^{-\frac{1}{2}}}{e^x}}$$

$$\lim_{x\to\infty} \frac{\sqrt{1+x^4}}{\sqrt{x^4}} = \lim_{x\to\infty} \sqrt{\frac{1+x^4}{x^4}}$$

$$= \lim_{x\to\infty} \sqrt{\underbrace{\frac{1}{x^4}}_{0}+1} = 1$$

compare with $x^2 = \sqrt{x^4}$

compare e^x, x^2 – slower than e^x

$$\lim_{x\to\infty} \frac{x^2}{e^x} = \lim_{x\to\infty} \frac{2x}{e^x} = \lim_{x\to\infty} \frac{2}{e^x} = 0$$

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31. $\sqrt{x}$ $\sqrt{10x+1}$ $\sqrt{x+1}$

same

same

$$\lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x+1}} = \lim_{x\to\infty} \sqrt{\underbrace{\frac{x}{x+1}}_{\to 1}} = 1$$

$$\lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{10x+1}} = \lim_{x\to\infty} \sqrt{\underbrace{\frac{x}{10x+1}}_{\frac{1}{10}}} = \sqrt{\frac{1}{10}}$$

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33.

$$\lim_{x\to\infty} \frac{\sqrt{9^x + 2^x}}{\sqrt{9^x - 4^x}}$$

$$3^x = \sqrt{(3^x)^2}$$
$$= \sqrt{3^{2x}}$$
$$= \sqrt{(3^2)^x}$$
$$= \sqrt{9^x}$$

$$\lim_{x\to\infty} \frac{\sqrt{9^x+2^x}}{3^x}$$ same

$$\lim \frac{\sqrt{9^x+2^x}}{\sqrt{(3^x)^2}} = \lim_{x\to\infty} \sqrt{\frac{9^x+2^x}{9^x}}$$

$$= \lim_{x\to\infty} \sqrt{1+\left(\tfrac{2}{9}\right)^x}$$

$\left(\tfrac{2}{9}\right)^x \to 0$

$= 1$

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8.4 a Improper Integrals

$$\int_1^\infty \frac{1}{x^2}\,dx$$

improper because of ∞ as a limit of intrgration

Converges to 1

$$\lim_{b\to\infty} \int_1^b \frac{1}{x^2}\,dx \stackrel{?}{=} 1$$

$$\int_1^b x^{-2}\,dx = \frac{x^{-1}}{-1} = -\frac{1}{x}\Big|_1^b = -\frac{1}{b} - -1$$

$$\lim_{b\to\infty} 1 - \frac{1}{b} = 1$$

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divergES

$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty}\int_1^b x^{-\frac{1}{2}}\,dx$$

$$\lim_{b\to\infty} 2x^{\frac{1}{2}}\Big|_1^b$$

$$\lim_{b\to\infty}\left(2\sqrt{b}-2\sqrt{1}\right)$$

$$=\infty$$

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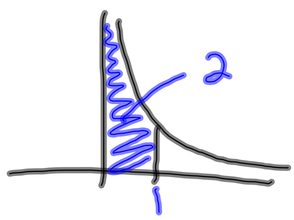
other types of improper integrals

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx$$



make it proper

$$\lim_{b\to 0^+}\int_b^1 x^{-\frac{1}{2}}\,dx$$

$$\lim_{b\to 0^+}\left(2x^{\frac{1}{2}}\Big|_b^1 = \lim_{b\to 0^+}\left(2-2\sqrt{b}\right)\right.$$

$$=2$$

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$$\int_{-1}^{1} \frac{1}{x^2}\,dx$$

$$\lim_{b\to 0^-}\int_{-1}^{b} \frac{1}{x^2}\,dx + \lim_{b\to 0^+}\int_{b}^{1} \frac{1}{x^2}\,dx$$

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